

# Assessing Throughput and Reliability in Communication and Computer Networks

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**Key Words** — Network reliability, Network performance, Stochastic model

**Reader Aids** —

**Purpose:** Widen the state of the art

**Special math needed for explanation:** Elementary stochastic processes, Applied queueing

**Special math needed to use the results:** None

**Results useful to:** Network reliability analysts and theorists

**Abstract** — System performance and reliability are jointly assessed for highly reliable communication/computer networks. The model assumes that at most a small number of components can be down at a time and that the average repair/replacement time of a failed component is small when compared with the average failure times of network components. The system performance is measured in terms of network throughput at steady-state operation.

## 1. INTRODUCTION

Perhaps the most important factors in judging the quality of a communication or computer network are its performance and reliability. If two systems have the same performance characteristics, one anticipates that in the long run more reliable systems will perform better than less reliable ones. However, when comparing two (or more) systems with both different performance characteristics and different reliabilities (availabilities), it is not always clear which system will be superior overall.

This paper concentrates on jointly evaluating performance and reliability for a communication/computer network in which 1) the performance criterion is system throughput, and 2) the network components are themselves highly reliable. The approach to this problem follows the methodology recently developed in [2]. A brief overview, a history of network reliability, and a short bibliography are also in [2]. It is assumed that the traffic flow in the network can be modeled by a network of Markov queues and that there is only one type of traffic (for instance, data packets).

The motivation and system description are described in the section 2. Section 3 describes the methodology to calculate the average throughput and the availability of a network with highly reliable components. Section 4 briefly discusses how the steady-state network throughput is obtained. Section 5 contains a few examples and section 6 is the concluding remarks.

## 2. SYSTEM DESCRIPTION & ASSUMPTIONS

In communication/computer networks, the failures of some components (nodes or links) can result in degraded performance of the network. The network fails if it reaches a state where its performance is not acceptable. Network failures usually have two failure modes:

- Connective failure, where failures of some components result in some disconnected nodes.
- Congestive failure, where the failures of some components result in overload and congestion in remaining components, blocking of the incoming traffic, buffer overflow, etc.

### Assumptions

- The network components are highly reliable; component MTTFs are large (order of magnitude: years or months, say). The failure times are exponentially distributed.
- The repair times are much shorter than MTTF (order of magnitude: days or hours).
- The call/packet/message interarrival times and processing times are much shorter than MTTR (order of magnitude: seconds or microseconds).
- The network is in a steady state; when a failure occurs, the traffic flow in the remaining working components reaches steady-state quickly (order of magnitude of this time to reach steady-state: seconds or minutes).
- The repaired components are as good as new.
- Components have 2 states: up and down.

The assumptions are not very restrictive, fit a large number of real networks, and render the problem tractable.

## 3. THE MODEL

### Notation

$N$	number of components in the network
$I$	number of nodes in the network
$\lambda_i$	failure rate, $i = 1, \dots, M$
$M$	number of distinct failure types
$D_i$	downtime due to failure type $i$
$p_i$	steady state probabilities, $i = 0, \dots, M$
$r(k)$	throughput of the network while the network state is $k$ : $k = 0, \dots, M$
AT	average network throughput
AV	network availability

Consider a network having  $N$  components (nodes + links). The components are highly reliable. From time to time, however, they fail. Sometimes even a group of components fails simultaneously. When a component(s) fails, a repair/replacement procedure is initiated. After repair, the component(s) is back to full function. The downtimes are relatively short when compared with failure times (up-times). Therefore, for simplicity, we assume that the probability of another component(s) failure during the downtime is zero. More precisely, we assume that a component failure (or simultaneous failure of a group of components) inhibits any future failures till the completion of the repair. There are  $M$  distinct failure types (states), each state having a positive probability of occurrence.

Under these assumptions, the behavior of the network can be modeled by the following stochastic process  $X(t)$ : Initially, the process  $X(t)$  spends an exponential amount of time (with the mean  $1/\lambda$ ) at state 0, the state in which all the network components are up. When a failure occurs, with the probability  $\lambda_i/\lambda$  it is of the type  $i$ :  $i = 1, \dots, M$ ; ( $\lambda = \sum_{i=1}^M \lambda_i$ ,  $\lambda_i > 0$ ). The downtime due to the failure type  $i$ ,  $D_i$ , has a general distribution. After completion of the repair, the process always returns to state 0 and the process oscillates between the up state 0 and some down state  $i$  (see figure 1). Equivalently: When the system returns to the state 0, then  $M$  independent Poisson processes start simultaneously.

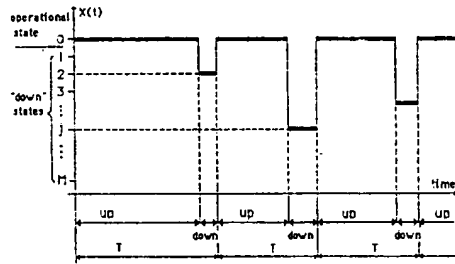


Fig. 1. A Typical Realization of the Process  $X(t)$

Process  $i$  corresponds to failure type  $i$  and has a failure rate  $\lambda_i$ . When the first failure occurs then, with the probability  $\lambda_i/\lambda$ , it is of the type  $i$  and the system enters repair stage  $i$  during which no new failures can be generated.

The instants at which  $X(t)$  changes from state  $i$  to state 0 can be regarded as the regeneration points of the process. It follows from the theory of regenerative processes (see Ross [3], or for more details [2]) that:

$p_j = \lim_{t \rightarrow \infty} \Pr\{X(t) = j\}$ ,  $j = 1, \dots, M$  the steady-state probabilities of the process  $X(t)$  are:

$$p_0 = 1 / \left( 1 + \sum_{j=1}^M p_j \right), \quad (3.1)$$

$$p_j = \rho_j \cdot p_0, \quad j = 1, \dots, M \quad (3.2)$$

$$\rho_j = \lambda_j \cdot E\{D_j\}, \quad j = 1, \dots, M. \quad (3.3)$$

Moreover, using theorem 3.6.1 [3, p 78], the average network throughput is:

$$\begin{aligned} AT &= \frac{E\{\text{throughput during one cycle}\}}{E\{\text{cycle time}\}} \\ &= \left[ r(0) + \sum_{k=1}^M r(k) \cdot p_k \right] / \left[ 1 + \sum_{k=1}^M p_k \right] = \sum_{k=0}^M r(k) \cdot p_k \end{aligned} \quad (3.4)$$

Cycle time is the time between two successive regeneration points.

$r(k)$ , the throughput at state  $k$ , is defined as the equilibrium throughput of the network while the network state is  $k$ .

Another measure of interest is the network availability. In this application the network is failed (down) if some of its nodes are disconnected or if the network is congested (not able to accept the entire arriving traffic). Network availability, in the usual manner, is:

$$AV = \frac{E\{\text{network uptime during one cycle}\}}{E\{\text{cycle time}\}} \quad (3.5)$$

Let  $I(k)$  and  $J(k)$  be indicator functions:

$$I(k) = \begin{cases} 1, & \text{network being in the state } k \text{ is connected,} \\ 0, & \text{otherwise;} \end{cases}$$

$$J(k) = \begin{cases} 1, & \text{network being in the state } k \text{ is not congested,} \\ 0, & \text{otherwise.} \end{cases}$$

It again follows from theorem 3.6.1 [3] that the network availability (3.5) is:

$$AV = \sum_{k=0}^M I(k) \cdot J(k) \cdot p_k. \quad (3.6)$$

#### 4. NETWORK THROUGHPUT MEASUREMENT

It is assumed that the traffic flow in the network in any state can be modeled by a network of Markov queues (for details see Kelly [1]).

##### Notation

When the network is in state  $k$ , ( $k = 0, 1, \dots, M$ ) —

- $\nu(k)$  network input traffic rate (offered traffic rate)
- $\nu^*(k)$  maximal network throughput
- $g_i(k)$  probability that a packet (call or message) appears initially at node  $i$ ;

$f_{ij}(k)$  probability that a packet finishing service at node  $i$  goes next to node  $j$ ;

$1 - \sum_j f_{ij}(k)$  probability that a packet leaves the network at node  $i$ ;

$\mu_i(k)$  service rate at node  $i$ ;

for all nodes  $i, j = 1, \dots, I$ .

Schweitzer [4] has shown that the maximal throughput  $\nu^*(k)$  through the network (being at state  $k$ ) is:

$$\nu^*(k) = [\max(a_1(k), \dots, a_I(k))]^{-1} \quad (4.1)$$

$$a_i(k) \equiv e_i(k)/\mu_i(k), \quad i = 1, \dots, I \quad (4.2)$$

$e_i(k)$ ,  $i = 1, \dots, I$  is the unique solution of the system of equations

$$e_i(k) = g_i(k) + \sum_{j=1}^I e_j(k) \cdot f_{ji}(k), \quad i = 1, \dots, I. \quad (4.3)$$

In matrix notation, when we denote  $\mathbf{g}(k) = (g_1(k), \dots, g_I(k))^T$ ,  $\mathbf{e}(k) = (e_1(k), \dots, e_I(k))^T$  and  $\mathbf{F}(k) = (f_{ij}(k))_{i,j=1,\dots,I}$  the system (4.3) can be written as:

$$\mathbf{e}(k) = \mathbf{g}(k) + \mathbf{F}^T(k) \cdot \mathbf{e}(k) \quad (4.3a)$$

and has the solution:

$$\mathbf{e}(k) = (\mathbf{I} - \mathbf{F}^T(k))^{-1} \mathbf{g}(k). \quad (4.4)$$

One can then define  $r(k)$ , the throughput of a network in state  $k$ , as

$$r(k) \equiv \min\{\nu(k), \nu^*(k)\}. \quad (4.5)$$

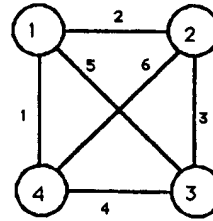
The index for which  $\nu^*(k) = a_j(k)^{-1}$  identifies the bottleneck node.

## 5. EXAMPLE

For clarity and simplicity in the calculations, consider a small symmetric network layout with 4 nodes and 6 links, as shown in figure 2a. Only links can fail, and there are 6 possible failure states — state  $i$  ( $i = 1, \dots, 6$ ), is defined as the state in which the link  $i$  failed.

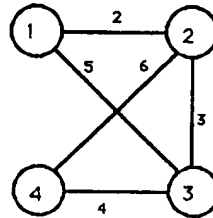
Let  $\lambda_i = 1/\text{year}$  and  $E\{D_i\} = 2/365$  years, for all  $i = 1, \dots, 6$ . Then  $p_0 = 0.968$ , and  $p_1 = p_2 = \dots = p_6 = 0.0053$ . Select  $\mathbf{g}^T(k) = (0.25, 0.25, 0.25, 0.25)$ ,  $\mu_1(k) = 0.5$ ,  $\mu_2(k) = 0.6$ ,  $\mu_3(k) = 0.7$  and  $\mu_4(k) = 0.8$  for all  $k = 0, 1, \dots, 6$ . The node processing rates are measured in (1/msec). When the link connecting node  $j$  fails, the link  $i$  traffic will be equally shared by the two remaining links connecting the node  $i$  with the rest of the network.

When the network is in state 0, then  $e_1(0) = e_2(0) = e_3(0) = e_4(0) = e$  and the system (4.3) reduces to one equation:  $e = (1/4) + (1/6) \cdot (e + e + e)$ . Solving this gives  $e = 0.5$ , and —



$$\mathbf{F}(0) = \begin{bmatrix} 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 \end{bmatrix}$$

(a) In the state 0 (fully operational state)



$$\mathbf{F}(1) = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix}$$

(b) In the state 1 (link #1 is down)

Fig. 2. Network Layout and the Routing Matrix

$$\begin{aligned} \nu^*(0) &= \left[ \max \left\{ \frac{1}{2\mu_1(0)}, \frac{1}{2\mu_2(0)}, \frac{1}{2\mu_3(0)}, \frac{1}{2\mu_4(0)} \right\} \right]^{-1} \\ &= [\max(1, 0.83, 0.71, 0.61)]^{-1} = 1 \end{aligned}$$

When the network is in state 1 [see figure 2b], the system (4.3) becomes —

$$e_1(1) = \frac{1}{4} + \frac{1}{6} (e_2(1) + e_3(1))$$

$$e_2(1) = \frac{1}{4} + \frac{1}{4} e_1(1) + \frac{1}{6} e_3(1) + \frac{1}{4} e_4(1)$$

$$e_3(1) = \frac{1}{4} + \frac{1}{4} e_1(1) + \frac{1}{6} e_2(1) + \frac{1}{4} e_4(1)$$

$$e_4(1) = \frac{1}{4} + \frac{1}{6} (e_2(1) + e_3(1))$$

Because of symmetry, we must have  $e_1(1) = e_4(1) = e$  and  $e_2(1) = e_3(1) = f$ , and it is easy to see that  $e = 7/16$  and  $f = 9/16$ . Thus  $\mathbf{e}^T(1) = (7/16, 9/16, 9/16, 7/16)$  and  $\nu^*(1) = [\max(0.8750, 0.9375, 0.8036, 0.7031)]^{-1} = 1.07$ .

Analogously we get:

$$e^T(2) = (7/16, 7/16, 9/16, 9/16); \quad \nu^*(2) = 1.14$$

$$e^T(3) = (9/16, 7/16, 7/16, 9/16); \quad \nu^*(3) = 0.88$$

$$e^T(4) = (9/16, 9/16, 7/16, 7/16); \quad \nu^*(4) = 0.88$$

$$e^T(5) = (7/16, 9/16, 7/16, 9/16); \quad \nu^*(5) = 1.07$$

$$e^T(6) = (9/16, 7/16, 9/16, 7/16); \quad \nu^*(6) = 0.88$$

In the failure states 1, 2, 5, the maximum throughput rate is higher than the maximum throughput rate in the fully operational network. This paradox can be explained by the fact that the selection of alternative routings puts a lesser load on the slowest nodes, thus creating favorable conditions for better throughput.

Let the offered traffic rate of the network  $\nu(k) = \nu = 0.95$  for all  $k = 0, 1, \dots, 6$ . The states with unacceptable performance are the states 3, 4, 6. Using (3.4) and (4.5), the average throughput of the network is —

$$AT = 0.95(p_0 + p_1 + p_2 + p_5)$$

$$+ 0.88(p_3 + p_4 + p_6) = 0.9486.$$

In this example we have only the congestive type of failures and therefore the connectivity failure indicator function  $I(k) = 1$  for all  $k = 0, 1, \dots, 6$ . The indicator function for congestive failure is —

$$J(k) = \begin{cases} 1, & \text{for } k = 0, 1, 2 \text{ and } 5; \\ 0, & \text{for } k = 3, 4, \text{ and } 6. \end{cases}$$

Thus the availability of our network is:

$$AV = p_0 + p_1 + p_2 + p_5 = 0.9839.$$

### SUMMARY

In this application, throughput rate is a measure of performance of a communication/computer network. The throughput and availability are assessed for a highly reliable network.

The assessment produces a single figure of merit and can be a valuable tool for network designers and network operations managers because it helps to evaluate potential measures for improvements, suggests routing alternatives, and estimates important parameters for both new and existing networks. However, the design of a communication/computer network is a very complex task which involves a wide range of factors. The designer is facing many objectives, some of which can be contradictory. The present model is just one building block of an entire decision-support system.

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